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B. E. (Third Semester) Examination, April-May 2020/ NOV-DEC 2020

(New Scheme)

(CSE Engg. Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

***Note :** Part (a) of each question is compulsory. Attempt any two part from (b), (c) and (d) of each question.*

1. (a) Define Fourier Series and write Euler's formula for Fourier series. 2

(b) Prove that

$$x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi.$$

Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}.$$ 7

(c) Obtain a half range cosine series for

$$f(x) = \begin{cases} Kx & , \quad 0 \le x \le l/2 \\ K(l-x) & , \quad l/2 \le x \le l \end{cases}$$ 7

and deduce the sum of the series.

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty$$ 7

(d) The following table give variations of periodic current over a period :

t (sec) :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp) :	1·98	1·30	1·05	1·30	–0·88	–0·25	1·98

Show that there is a direct current part of 0·75 amp in the variable current and obtain the amplitude of the first harmonic. 7

2. (a) Write down an expression for the Laplace transforms of periodic function $f(t)$ with period T. 2

(b) Find the Laplace transform of : 7

(i) $f(t) = \dfrac{e^{-at} - e^{-bt}}{t}$

(ii) $f(t) = |t-1| + |t+1|, \; t \ge 0$

(c) Use convolution theorem to evaluate :

$$L^{-1}\left[\frac{S^2}{(S^2+a^2)(S^2+b^2)}\right]$$ 7

(d) Solve

$$\frac{d^2x}{dt^2} + 9x = \cos 2t,$$

if $x(0) = 1, x(\pi/2) = -1$. 7

3. (a) State Cauchy's Residue theorem. 2

(b) If $f(z)$ is a regular function of z, prove that :

$$\nabla^2 |f(z)|^2 = 4|f'(z)|^2.$$ 7

(c) Find the Laurent'z series expansion of

$$f(z) = \frac{7z-2}{(z+1)z(z-2)}$$

in the region $|<z+1<3|$. 7

(d) Apply calculus of Residues to prove that :

$$\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2} = \frac{2\pi}{1-p^2} \quad (0<p<1)$$ 7

4. (a) From the partial differential equation by eliminating the arbitrary functions from

$$z = f(x+at)+g(x-at)$$ 2

(b) Solve $(mz-ny)\frac{\partial z}{\partial x}+(nx-lz)\frac{\partial z}{\partial x} = ly-mx$. 7

(c) Solve

$$\left(D^2-DD'-2D'^2\right)z = (y-1)e^x.$$ 7

(d) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t}+u, \text{ given that } u(x,0)=6e^{-3x}.$$ 7

5. (a) Define Probability Density Function. 2

(b) The probability Density $p(x)$ of a continuous random variable is given by

$$p(x) = y_0\, e^{-|x|}, \quad -\infty<x<\infty$$

Prove that $y_0 = \frac{1}{2}$. Find Mean and Variance. 7

(c) The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that

(i) exactly two will be defective

(ii) at least two will be defective

(iii) none will be defective 7

(d) Fit a Poission distribution to the set of observations :

x :	0	1	2	3
f :	122	60	15	2

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